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Combined Optimal Price and Optimal Inventory Ordering Policy with Income Elasticity

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ABSTRACT. In the present paper, the effect of income elasticity on demand has been considered and the buyer's optimal special order quantity and optimal time has been determined, when supplier reduces sale price then buyer may offer a discount to push his sale to increase his profit margin. Income elasticity effect in inventory analysis has been introduced to obtain the gain equation to find the optimal special order quantity and profit associated with it when remnant inventory is zero. The profit and optimal order quantity are also derived when remnant inventory is finite. The cost of saving was also determined when due to inflation rise in the sale price is imminent. Finally optimal inventory ordering policy has been discussed to get maximum profit.

1. Introduction

The Elasticity of demand is generally associated with the name of Alfred Marshall Elasticity of demand refers to the responsiveness or sensitiveness of demand to change in price. But, when demand is inelastic, even drastic change in price may not affect the quantity demanded in any appreciable manner: for example the "Giffen Paradox" is that "a fall in price (Giffen Goods) tends to reduce demand and a rise in its price tends to extend its demand".

A new concept of income elasticity vis-a-vis inventory models has been introduced in this article. Income elasticity demand may be measured as under :

Ei = Proportionate change in the quantity demanded / Proportionate change in the income.

The optimal policies for no buyer's discount and constant demand are derived in Ardalan[1]. Optimal ordering policies in response to a permanent price increase when demand is assumed to be constant have been well documented by

Ardalan[2], Brown[3] and Taylor[10]. A model to determine the optimal ordering policies for a finite horizon consisting of two distinct time intervals characterised by different inventory parameter has been presented by Lev[6]. Models which determine the optimal special order quantity when a supplier reduces his price temporarily assume that the reduce price is in effect at the buyer's replenishment time and increase in demand. Ardalan[4] relaxes demand assumption made in most inventory system with price change.

The present paper considers the effect of income elasticity on demand and determines the buyer's optimal special order quantity and optimal time. When a supplier reduces the price temporarily, the buyer may offer a discount to his customer to push his sale and increase his profit margin. This paper brings into picture income-demand relationship which deals with classification of products into essential and non-essential goods. For non-essential goods like luxuries the income-demand relationship is considerable. The income elasticity effect into inventory analysis has been introduced in this paper and the gain equation has been obtained to find optimum special order quantity and profit associated with it when remnant inventory is zero. The optimum order quantity was also determined when there was finite remnant inventory and was extended to find the optimal time for placing such an order. The model was further modified to find the cost of saving of special purchases which a supplier announces to increase his sale price by some amount from a specified date. Finally optimal inventory ordering policy has been discussed to get maximum profit.

2. Model Assumptions

The model has been derived and analysed by taking the following assumptions into consideration :

1. The demand has been assumed as income-dependent and is known. A price reduction is accompanied by increase in demand and simultaneously effect of income elasticity has also been considered (Proportionate increase in income is followed by proportionate increase in demand).
2. The model is flexible in its assumption and can be suitably used to a scenario in which effect of income elasticity is insignificant (*i.e.* there is price elasticity).
3. The lead time has been taken to be zero to make presentation of the model simple and lucid, however, positive lead time can be easily introduced.
4. There are no shortages and demand is overcome from the remnant inventory.
5. Quantity discount is not available, and unit price is constant except when reduced price/increased price is in effect.
6. The total inventory cost during any interval is equal to long run average total cost multiplied by the length of the interval.

7. The supplier's reduced price/increased price is very short and ordering cost is relatively high so that it is not economical to place an order to just cover the demand during the fluctuation in price offered by the supplier.

The parameters used in the model has been defined as follows :

Ei = Proportionate change in demand/Proportionate change in income.

D = Buyer's annual demand.

D_1 = Buyer's annual demand after discount to customer.

D_2 = Buyer's new annual demand after the effect of income elasticity.

Thus, new annual demand = Old demand + Change in demand

$D_2 = D_1 + (Ei \times \text{proportionate change in income} / 100)\, D_1$.

$$D_2 = D_1 (1 + Y), \text{ where } Y = (Ei \times \text{proportionate change in income} / 100). \quad (1)$$

P = Supplier's regular price per unit.

P_1 = Buyer's regular price per unit.

P_2 = Buyer's sale (reduced) price per unit.

C = Ordering cost per order.

H = PF, F = Annual holding cost as a fraction of unit cost (Carrying cost parameter).

E.O.Q. = The size of an order that minimises the total inventory cost is known as economic order quantity.

Q = Lot size or order quantity in units (a special order quantity).

TC = Total annual cost.

Q_r = Regular optimal order quantity: $Q_r = [2CD / PF]^{1/2}$ (2)

Further if, "d" is the amount which supplier offers on his sale price, and Q_d is regular optimal order quantity using the reduced price, and if income elasticity effect is there, then :

$$Q_d = [2D_2 C/(P - d) F]^{1/2} = [2D_i(1 + Y) C/ (P - d) F]^{1/2} \quad (3)$$

If however the income elasticity effect is not there then

$$Y = O ; Q_d = [2D_1 C/(P - d) F]^{1/2} \quad , \quad \text{where} \quad (4)$$

Q_0 = Special optimal order quantity when remnant inventory is Zero.

Q_q = Special optimal order quantity when remnant inventory is not Zero.

G_r = Gain associated with regular policy during Ts.

G_s = Gain associated with special order during Ts.

T_s = The time interval between the time the buyer receives the special order of size 'Q' and his next replenishment time.

ts = The time when supplier's sale period starts.

tf = The time when supplier's sale period finishes.

tr = The time when the buyer replenishes his order.

Case 1. *Optimal special order quantity when remnant inventory is zero and effect of income elasticity is significant.*

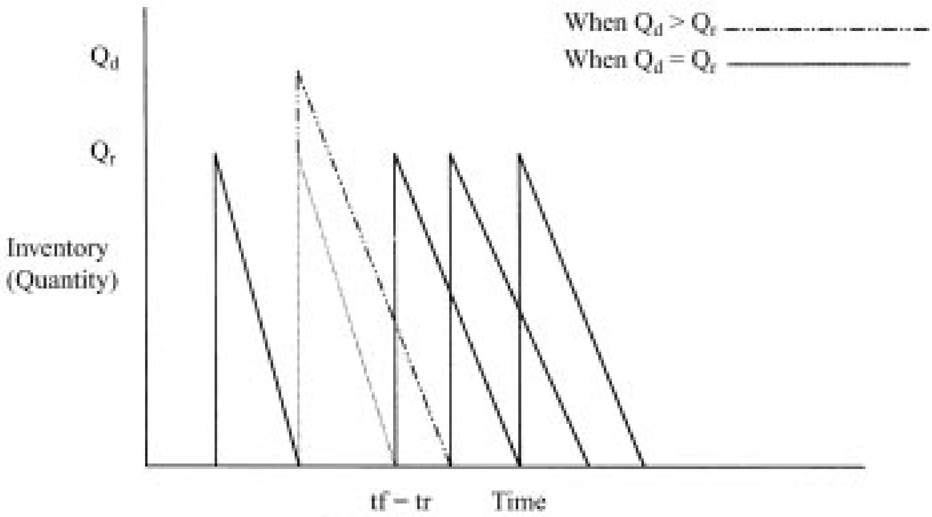

FIG. 1. Special order quantity when (tf = tr) remnant inventory is zero.

Suppose D_2 is the buyer's new annual demand after change in income elasticity of customer. At replenishment time buyer places an order of Q units at reduced price (offered by supplier). The buyer too offers a discount to his customers on all the Q units. T_s is the time interval after which the supplier reduced price is not available and buyer reverts to his usual ordering policy.

The gain associated within the time interval T_s is then given by :

$$G_s = [P_2 - (P - d)] Q - [Q^2 (P - d) F] /2D_1 (1 + Y)] - C \tag{5}$$

Suppose if 'd' is the discount offered on Q_d units, then purchase cost of these Q_d units would be :

$$(P - d)\ Q_d$$

and the number of units bought at usual (regular) price during would be :

$$T_s = [D/D_1\ (1 + Y)]\ (Q - Q_d) \quad (6)$$

Therefore, the total buying cost would be :

$$(P - d)\ Q_d + [PD/D_1\ (1 - Y)]\ (Q - Q_d) \quad (7)$$

The holding cost of the first order would be :

$$[Qd^2\ (P - d)\ F] / [2D_1\ (1 - Y)] \quad (8)$$

The holding cost during the rest of Ts would be :

$$[Q_r\ (Q - Q_d)\ PF] / [2D_1\ (1 + Y)] \quad (9)$$

The total holding cost during T_s, would be :

$$[Q_d^2\ (P - D)\ F] / [2D_1(1 + Y) + [Q_r(Q - Q_d)\ PF] / [2D_1(1 + Y)] \quad (10)$$

The number of orders placed during T_s, would be :

$$[1 + D(Q - Q_d)] / [D_1(1 + Y)\ Q_r] \quad (11)$$

The ordering cost associated with this order will be given by

$$C[1 + D(Q - Q_d)] / [D_1(1 + Y)\ Q_r] \quad (12)$$

The gain associated with the usual (regular) ordering policy during Ts, would be given by :

$$G_r = [P_2 - (P - d)]\ Q_d + [(P_1 - P)\ D(Q - Q_d)] / [D_1(1 + Y)] - [(P - d)$$
$$FQ_d^2] / [2D_1(1 + Y)] - [Q_r(Q - Q_d)$$
$$PF] / [2D_1(1 + Y)] - C[1 + D(Q - Q_d)] / [D_1(1 + Y)\ Q_r] \quad (13)$$

In order to maximise gain, the difference between Gr and Gs should be maximised.

$$G = [P_2 - (P - d)]\ Q - [Q^2(P - d)\ F] / [2D_1(1 + Y)] - C - [P_2 - (P - D)]$$
$$Q_d - [(P_1 - P)\ D\ (Q - Q_d)] / D_1(1 + Y) + [(P - d)\ FQ_d^2] / [2D_1(1 + Y)]$$
$$+ [Q_r(Q - Q_d)\ PF] / [2D_1(1 + Y)] + C[1 + D(Q - Q_d)] / [D_1(1 + Y)\ Q_r] \quad (14)$$

Differentiating G w.r.t Q and equating it to zero, the following is obtained :

$$Q_0 = [D_1(1 + Y)\ (P_2 - P + d) - (P_1 - P)\ D] / [(P - d)\ F] + [Q_rP / (P - d)]$$

Putting Q_0 in Eq. (14) the optimal value of G is obtained which is as follows :

$$G = C[Q_0 - Q_d]^2 / Q_d^2$$

The outcome of model is illustrated by a numerical example given below :

Example 1

The per capita income of a country rises with 1% growth income, and the income elasticity is 5.0. The annual demand for a product is 10,000 units when the unit price is \$13.0. The ordering cost per order is \$10.0 and inventory holding cost is charged at 0.25 units purchase cost. This product is regularly purchased at \$10.0 per unit. The supplier is offering a discount of \$2.0 per unit for a short time. There are no stocks (*i.e.* remnant inventory is zero) and the buyer decides to replenish his inventory and offer a discount of \$1.0 to his customers and sell the units at \$12.0 per unit. The annual demand at this price is 13,000 units.

Solution

$$D_2 = 13{,}000\,(1 + 5 \times 1/100) = 13{,}650 \text{ units.}$$

$$Q_r = [(2 \times 10{,}000 \times 10) / (0.25 \times 10)]^{1/2} = 282.84 \text{ units.}$$

$$Q_d = [(2 \times 13650 \times 10) / (0.25 \times 8)]^{1/2} = 369.45 \text{ units.}$$

$$Q_0 = [(13650)\,(12 - 10 + 2) - (10000)\,(13 - 10)] / [(0.25)\,(10 - 2)] + (282.84 \times 10) / 10 - 2) = 12653.55 \text{ units.}$$

The gain associated with special order is given by

$$G = 10[(12653.55 - 369.45) / 369.45]^2 = \$11{,}054.86$$

Case 2: *Optimal special order quantity when remnant inventory is not zero, and effect of income elasticity is significant.*

If effect of income elasticity is considerable and there is sufficient remnant inventory, then buyer has two options – either to buy at the reduced price of the supplier and subsequently offer discount to his customer or just avoid the business ploy and follow his usual ordering policy.

If ‘q is the level of remnant then to determine special order quantity the difference between G_r and G_s should be maximised. If per capita income rises causing rise in purchasing power of customer, the gain associated with special order quantity (G_s) and gain associated with usual ordering policy (G_r) are expressed as :

$$G_s = Q(P_2 - P + d) - [q^2PF] /2D - [qQ(P - d) F] /D - [P - d) FQ^2] / [2D_1(1 + Y)] - C \tag{15}$$

$$G_r = [DQ(P_1 - P) / D_1(1 + Y)] - [q^2PF] /2D - [Q_rQPF] / [2D_1(1 + Y)] - [DQC] / [D_1(1 + Y)\, Q_r] \tag{16}$$

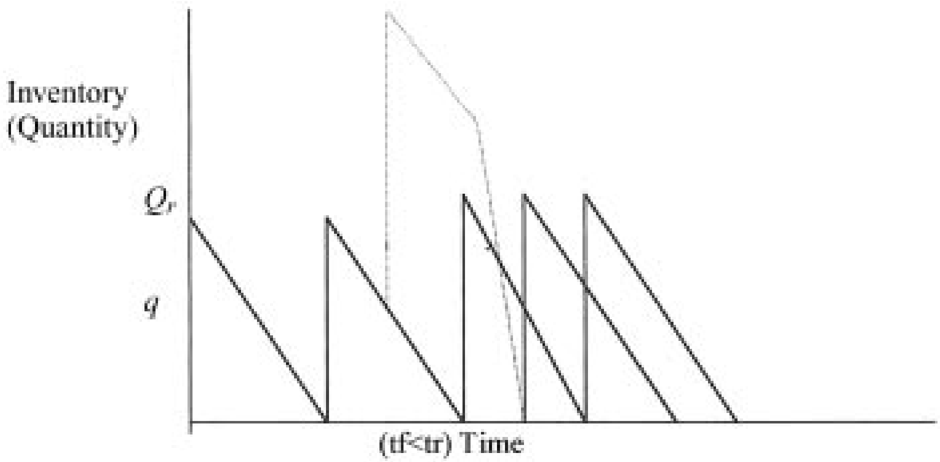

FIG. 2. Special order when remnant (if < tr) inventory is not zero.

The increase in gain due to special order is G = Gs – Gr

$$G = Q(P_2 - P + d) - [q^2PF] / [2D] - [(Qq(P - d) F)] / D - [(P - D) FQ^2] / [2D_1(1 + Y) - C - [DQ(P_1 - P)] / [D_1(1 + Y)] + [q^2PF] / [2D] + [Q_rQPF] / [2D_1(1 + Y) + [DQC] / [D_1(1 + Y) Q_r] \quad (17)$$

To maximise profit differentiating G.w.r.t Q and equating it to be zero, it would be :

$$Q_q = [(P_2 - P + d) D_1(1 + Y) - D(P_1 - P)] / [(P - d) F] + Q_rP / (P - d) F] - [qD_1(1 + Y) / D \quad (18)$$

$$Q_q = Q_0 - [QD_1(1 + Y) / D]. \quad (19)$$

Example 2

Consider the same *Example 1* and consider that there are 100 units in the stock when the supplier's reduced price ends.

Solution

The usual (regular) optimal order quantity will be given by :

$$Q_r = [2 \times 10000 \times 10/.25 \times 10]^{1/2} = 282.84 \text{ units.}$$

When remnant inventory is zero then special order quantity would be :

$$Q_0 = 12653.55 \text{ units.}$$

The special order quantity when the remnant inventory is 100 units *i.e.* $q = 100$ would be :

$$Q_q = 12517.05 \text{ units.}$$

The increase in gain due to this special order is

$$G = \$11438.06$$

To determine the optimal time for placing special order Q_q, a theorem is deduced as follows :

Theorem 1

The higher the income elasticity, the smaller the level of remnant inventory and it determines the optimal time for placing order.

Proof

Proof is provided in Appendix A(a).

Theorem 2

The higher the income elasticity, the larger the level of remnant inventory, and higher is the cost of saving when price rise is imminent.

Proof

Proof is provided in Appendix A(b).

The following example illustrates the technique used :

Example 3

The data of *Example 1* is taken, but now supplier announces price increase in his units from \$10.0 to \$11.0 on 1st of April. What should be the special order quantity on 31st of March before price increase is effective if the remnant inventory is 100 units. What will be the cost of saving when income elasticity is high?

Solution

$$D^* = 10000\,(1 + 5 \times 1/100) = 10500 \text{ units.}$$

The optimal special order size $Q_{so} = 4251.99$ units.

The optimal cost of saving associated with this order is, $G = \$2092.44$

Theorem 3

If tf < tr, the optimal order policy is

1) To order Q_q at 'tf' of $Q_q > Q_d$ otherwise
2) Order Q_r at 'tr'

Proof

Proof is provided in Appendix A(c).

3. Numerical Results and Discussion

The price, the order quantity, and the replenishment time that maximize total profit are chosen as the optimal sale price and optimal inventory policy. The combination of the procedure and the model presented here determines the buyers optimal reduce price and the optimal inventory policy in response to a sale by a supplier under the effect of income elasticity.

The combined optimal price and optimal order quantity occurs at $12.65 per unit and 10,934.29 units respectively by Ardalan[2], who considers income-demand relationship. When income elasticity is taken into consideration then combined optimal price and optimal order quantity occurs at $12.35 per unit and 12,263.73 units at 1% growth of income, at $12.15 per unit and 13,687.66 units at 2% growth of income, at $12.00 per unit and 15,253.52 units at 3% growth of income respectively. The Computer results are shown in Appendix B.

The combined optimal price and optimal order quantity when remnant inventory is finite occurs at $12.65 per unit and 10,934 units respectively by Ardalan[2]. When income elasticity effect is taken into consideration then combined optimal price and optimal order quantity occurs at $12.35 per units and 12,140.00 units at 1% growth of income when there is finite remnant inventory. At 2% and 3% growth of income the optimal price and optimal order quantity are $12.15 per unit, 13,551.11 and $12.05 per unit and 14,979.06 units respectively. The Computer results are shown in Appendix B.

4. Conclusion

The present paper considers the effect of income elasticity on demand and determines the buyer's optimal special order quantity and optimal time. In case I, the optimal special order quantity is determined when remnant inventory is zero. The gain associated with this special order quantity is derived. A numerical example is presented to illustrate the essential features of the method. In Case 2, the optimal special order quantity is determined when remnant inventory is finite. A numerical example is presented to find optimal order quantity and gain when remnant inventory is 100 units. To determine the optimal time for placing special order quantity, Theorem 1 is deduced. In Theorem 2, the model has been further modified to find the cost of saving when price rise is imminent. Finally optimal inventory ordering policy is discussed in Theorem 3.

To illustrate the application of the procedure presented in the model a Fortran Program is developed to solve the sample problem it was seen that at 1% growth of income the gain was maximum at $12.35 per unit and after this value the gain starts decreasing. The same pattern of trend was seen at 2% and 3% growth of income. Also when remnant was finite then gain was maximum at $12.35 per unit and after this value the gain started decreasing. The same pattern of trend was seen at 2% and 3% growth of income, where the maximum gain was at $12.15 and $12.05 per unit respectively.

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Appendix A

(a) Proof of Theorem I

When remnant inventory is 'q' the special order quantity is 'Q_q'

When remnant inventory is (q – 1) then optimal special order is given by

$$Q_q + D_1(1 + Y) / D$$

If Ei is income elasticity, then new demand due to it is D_2 given by

$$D_2 = D_1(1 + Y)$$

Using the gain Eq. (17), putting Q = Q_q when remnant inventory is 'q' units and gain G as Gq

And Q = Q_{q-1} when remnant inventory is (q – 1) units and gain G as G_{q-1}

Where, $Q_{q-1} = Q_q + [D_1(1 + Y) / D]$

$$\text{Eq (17)} => G_q = Q_q(P_2 - P + d) - [qQ_q(P - d) F] / D - [(P - d) FQq^2 / [2D_1(1 + Y)]$$
$$- C - [DQ_q(P1 - P)] / [D_1(1 + Y)] + [Q_rQ_qPF] / [2D_1(1 + Y) + [DQ_qC] / [D_1(1 + Y) Q_r] \quad (A.1)$$

For remnant inventory (Q_{q-1})

$$\text{Eq. (17)} => G_{q-1} = Q_{q-1}(P_2 - P + d) - [(q - 1) Q_{q-1}(P - d) F] / D - [(P - d)$$
$$F(Q_{q-1})^2] / [2D_1(1 + Y)] - C - [DQ_{q-1}(P_1 - P)] / [D_1(1 + Y)]$$
$$+ [Q_rQ_{q-1} PF] / [2D_1(1 + Y) + (DQ_{q-1}C] / [D_1(1 + Y) Q_r] \quad (A.2)$$

Here we need to prove Gq – 1 > Gq *i.e.* Gq – 1 – Gq = positive

$$G_{q-1} - G_q = [D_1(1 + Y) (P_2 - P + d)] / D - [D_1(1 + Y) q(P - d) F] / D^2$$
$$+[D_1(1 + Y) (P - d) F] /2D^2 - P_1 + P + [Q_rPF] / 2D \quad (A.3)$$

Using Eq. (18) in Eq. (A.3), we get

$$[G_{q-1} - G_q] [D/(P - d) F] = Q_q + D_1(1 + Y) /2D \quad (A.4)$$

Which is a positive value.

(b) Proof of Theorem 2

If the supplier changes his sale policy and he stops the discount given to a customer and announces a new increased sale price

Let p be the amount by which the supplier tends to increase his sale price at some date t_i. The units purchased before ti will cost 'p' but purchase after ti will cost (P + p).

If D is the annual demand and E_i is the income elasticity, 'q' is level of remnant inventory, then New Demand due to income elasticity = Old demand + change in demand.

$$D^* = D(1 + Y) \quad (A.5)$$

$$Q_r = [2CD^*/PF]^{1/2} = [2CD(1 + Y) / PF]^{1/2} \quad (A.6)$$

The optimal order quantity after the price rise will be,

$$Q_r^* = [2CD^* / (P + p) F]^{1/2} = [2CD(1 + Y) / (P + p) F]^{1/2} \quad (A.7)$$

$$TC_r = PQ_s + [Q_sPFq] / [D(1 + Y) + [Q_s^2PF] / [2D(1 + Y)] + [q^2PF] / [2D(1 + Y) + C \quad (A.8)$$

$$TC_s = ((P + p) Q_s +(1/2) [(2CD(1 + Y)) / ((P + p) F)]^{1/2} (P + p) F / Q_s / [D(1 + Y)]$$

$$+ [q^2PF] / [2D(1 + Y)] + [Q_s / Q_r^*]C \quad (A.9)$$

To find the optimal order size the difference between TC_r and Tc_s should be maximised

$$G = Q_s[p +[(CF(P + p) / (2D(1 + Y)]^{1/2} - PFq / D(1 + Y)] - [PFQ_s^2] / [2D(1 + Y)] - C \quad (A.10)$$

Differentiating w.r.t. Qs and equating it to zero to get Q_{so}

$$Q_{so} = Q_r^* / 2 + p / 2PF[2D(1 + Y) + Q_r^*F] - q \quad (A.11)$$

This is the optimal special order size, using Eq. (A.11) in Eq. (A.10) we get the optimal cost of saving associated with Q_{so}

$$G = Q_{so} [P +[CF(P + p)) / (2D(1 + Y))]^{1/2} - PFq / D(1 + Y)] - [PFQ_{so}^2] / [2D(1 + Y)] - C$$

(c) Proof of Theorem 3

The value of the remnant inventory Q_r for which G_r and G_s are equal can be determined by equating Eq. (15) and (16)

$$Q(P_2 - P + d) - [q^2PF] / 2D - [qQ(P - d) F] / D - [P - d) FQ^2] / [2D_1(1 + Y)]$$

$$- C = [DQ(P_1 - p) / D_1(1 + Y)] - [q^2PF] / 2D - [Q_rQPF] / [2D_1(1 +Y)]$$

$$- [DQC] / [D_1(1 + Y) Q_r] \quad (A.12)$$

Simplifying and putting Q = Qq and q = qc and using Eq. (18), we get

$$Q_q / 2 = Q_d^2 / 2Q_q = > Q_q^2 = Qd^2$$

Since $G_s > G_r$ for any $Q_q > Q_d$ as long as Eq. (A.12) holds good and optimal order size Qq at ‘tf’ would maximise the gain. Otherwise the usual (regular) ordering policy will be optimal.

Appendix B

To illustrate the application of the procedure presented above a Fortran program was developed to solve the sample problem. The data of Example 1 is used to compute the optimal order quantity and gain at different rates of growth of income.

TABLE 1. When remnant inventory is zero.

Reduced price	1% Growth of income		2% Growth of income		3% Growth of income	
	Optimal order quantity	Gain	Optimal order quantity	Gain	Optimal order quantity	Gain
12.45	12070.23	11446.63	12865.67	12664.85	13661.10	13912.95
12.40	12171.54	11465.88	13029.54	12770.43	13887.54	14108.75
12.35	12263.73	11468.60*	13182.66	12854.99	14101.60	14279.08
12.30	12346.79	11455.12	13325.04	12918.85	14303.29	14424.17
12.25	12420.72	11425.79	13456.66	12962.31	14492.60	14544.33
12.20	12485.53	11380.96	13577.53	12985.76	14669.53	14639.89
12.15	12541.22	11321.05	13687.66	12989.57*	14834.09	14711.21
12.10	12587.78	11246.44	13787.03	12974.16	14986.28	14758.74
12.05	12625.22	11157.55	13875.65	12939.98	15126.09	14728.91
12.00	12653.53	11054.82	13953.53	12887.47	15253.52	14784.20*
11.95	12672.77	11938.69	14020.65	12817.12	15368.59	14763.14
11.90	12682.77	10809.63	14077.02	12725.43	15471.27	14720.25

TABLE 2. When remnant inventory is not zero.

Reduced price	1% Growth of income		2% Growth of income		3% Growth of income	
	Optimal order quantity	Gain	Optimal order quantity	Gain	Optimal order quantity	Gain
12.45	11950.15	11883.08	12742.01	13120.53	13533.88	14387.00
12.40	12049.64	11900.90	12903.74	13225.81	13757.84	14583.83
12.35	12140.00	11901.87*	13054.71	13309.70	13969.42	14754.13
12.30	12221.24	11886.34	13194.94	13372.50	14168.63	14899.04
12.25	12293.35	11854.68	13324.41	13414.57	14355.47	15018.59
12.20	12356.33	11807.26	13443.13	13436.27	14529.93	15113.13
12.15	12410.19	11744.47	13551.11	13438.00*	14692.02	15183.06
12.10	12454.93	11666.73	13648.33	12420.20	14841.73	15228.81
12.05	12490.54	11574.45	13734.80	13383.31	14979.06	15250.83*
12.00	12517.03	11468.10	13810.53	13327.81	15104.02	15249.64
11.95	12567.02	11368.10	13920.51	13300.21	15129.02	15212.23
11.90	12603.26	11268.43	14211.62	12280.31	15300.56	15201.43

سياسة العروض المشتركة للكلفة المثلى والجرد الأمثل في ظل مرونة العائد

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المستخلص . تتناول الورقة مشكلة بيانات الجرد أحادية الفترة حيث يؤخذ الطلب كدالة غير خطية . يحتوي النموذج على المكونات : سعر الشراء ، القيمة التالفة وكذلك سعر النقصان . إضافة إلى ذلك فهناك سعر الوحدة لجهد التسويق وتوزيع الطلب الذي يزداد بزيادة الجهد . لقد تم افتراض إزاحة توزيع الطلب إلى أعلى بزيادة جهد المبيعات ، لذا تتناول الورقة التحديد الآني للطلب الكمي الأمثل وجهد المبيعات . إضافة إلى ذلك فقد تم تحديد الربح الأمثل في ظل جهد التسويق الأمثل . لقد تم إجراء تحليل الحساسية للنتائج وذلك للأخذ في الاعتبار التغيير في الكمية المثلى المطلوبة ، وجهد المبيعات مع التغييرات في كلفة الوحدة للمبيعات ، بالإضافة إلي كلفة الوحدة للمشتريات ، وكلفة القيمة التالفة ، وكلفة النقصان ، وكلفة ا لوحدة لجهد التسويق .