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Convergence theorems for ψ-expansive and accretive mappings

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Abstract

Let E be a real Banach space, and let $A : D(A) \subseteq E \to E$ be a Lipschitz, ψ-expansive and accretive mapping such that $\overline{co}(D(A)) \subseteq \cap_{\lambda>0} \mathcal{R}(I + \lambda A)$. Suppose that there exists $x_0 \in D(A)$, where one of the following holds: (i) There exists $R > 0$ such that $\psi(R) > 2\|A(x_0)\|$; or (ii) There exists a bounded neighborhood U of x_0 such that $t(x - x_0) \notin Ax$ for $x \in \partial U \cap D(A)$ and $t < 0$. An iterative sequence $\{x_n\}$ is constructed to converge strongly to a zero of A. Related results deal with the strong convergence of this iteration process to fixed points of ψ-expansive and pseudocontractive mappings in real Banach spaces. The convergence results established in this paper are new for this more general class of ψ-expansive and accretive or pseudocontractive mappings.

MSC: 47H04; 47H06; 47H30; 47J05; 47J25

Keywords: Accretive mapping; ψ-expansive mapping; Pseudocontractive mapping; Banach space

1. Introduction

Let E be a real normed linear space with dual E^*. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$Jx = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\},$$

where $\langle ., . \rangle$ denotes the generalized duality pairing. It is well known that if E^* is strictly convex then J is single-valued.

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