Folding-crumpling of thin-walled aluminium frusta

A. A. A. Alghamdi

Department of Mechanical Engineering, King Abdulaziz University, P.O. Box 80204, Jeddah 21589, Saudi Arabia

Abstract - This paper reports a new innovative mode of plastic deformation of specific thin-walled frusta when crushed axially between two rigid parallel plates with free-fixed boundary conditions. This folding-crumpling mode is only observed in frusta within a limited range of angles. Deformable energy absorbers are briefly introduced with special attention to frusta. Experimental results for the observed deformation mode of capped-end machined aluminium frusta under quasi-static testing condition are given. Obtained results show excellent absorbing capacity of the folding-crumpling mode when compared to the other modes reported in the literature such as crushing or inversion.

NOTATION

d	frustum small diamter
D	frustum large diamter
h	frustum height
m	frustum mass
t	wall thickness
α	angle of frustum

INTRODUCTION

Collapsible energy absorbers are systems that convert kinetic energy in crash events into irreversible permanent plastic deformation energy stored in deformable solids. The absorption process depends on factors such as, material of the absorber, magnitude of the load, method of application of the load, transmission rates and deformation displacement patterns [1].

Collapsible energy absorbers are used as crash protection devices working to minimize the potential of human injury and property damage during crashes. This is achieved by decreasing the impacting force so that the resulting stress is within the tolerance limit of the human body or the yield strength of the equipment. Crushable energy absorbers are used in daily life as road crash barriers, automobile bumpers, crash retards in emergency systems and so on.

Famous collapsible energy absorber units include cylindrical shells [2], square tubes [3], inverbucktubes [4], tube inversion [5] and frusta [6]. Circular shapes provide perhaps the widest range of all choices for use as absorbing elements because of their favourable plastic behaviour under axial forces, as well as their common occurrence as structural elements.

In this paper, the selected absorber has the shape of capped-end frustum, which is a truncated cone with upper closed end, see Figure 1. Frusta are used in industry, especially in the domains of missiles and armaments. The frustum is placed between two parallel plates and crushed axially with upper capped-end free and lower end constrained. Several frusta with different angles and thicknesses were crushed using this set up in an attempt to maximize the absorbed energy per unit mass, which is an acceptable objective among researchers in impact mechanics.

AXIAL CRUSHING OF FRUSTA

The behaviour of thin tubes when subjected to axial loads has been of particular interest since the pioneering works of Alexander in 1960 [2]. Johnson and Reid [7] identified the dominant modes of deformation in simple structural elements with different cross sectional areas when subjected to various quasi-static loadings. They described the load-deformation characteristics of a number of these elements such as circular tubes and w-shape structure. The study of deformation of axisymmetric energy absorbers falls into two main categories; axial and lateral loadings. In

comparing lateral with axial behaviour, the specific energy in the axial crushing mode was found to be ten times that of lateral mode [8].

Capped-end frusta are truncated circular cones with closed end, see Figure 1. Postlethwaite and Mills studied axial crushing of frusta in 1970 [9]. In their study, they used Alexander's model to predict the mean crushing force for the concertina mode of deformation for frusta made of mild steel.

Mamalis and Johnson (1983) investigated experimentally the crushing of aluminium frusta with large angles (α=80° and 85°) when subjected to axial compression load under quasi-static conditions with free-free boundary condition. Mamalis, Johnson and Viegelahn [10] extended their experimental study to include mild steel at elevated strain rates. Mamalis, Manolakos, Saigal, Viegelahn and Johnson [11] refined the work of Postlethwaite and Mills in using the extensible collapse analysis for predicting the mean crushing load for concertina mode of deformation. In another paper, Mamalis, Manolakos and Viegelahn [12] investigated the crumbling of PVC frusta into diamond mode of deformation. Mamalis, Manolakos, Viegelahn and Johnson [13] modelled the progressive extensible collapse of frusta and gave a theoretical model that depicts the changes in peaks and troughs of the experimental load-displacement curves.

Inversion of frusta was first reported by Alghamdi [14] and then investigated in details using finite element modelling [15]. Aljawi and Alghamdi [16] described dynamic inversion and later on, full study of inversion of frusta with large number of angles and wall thicknesses was carried out [17].
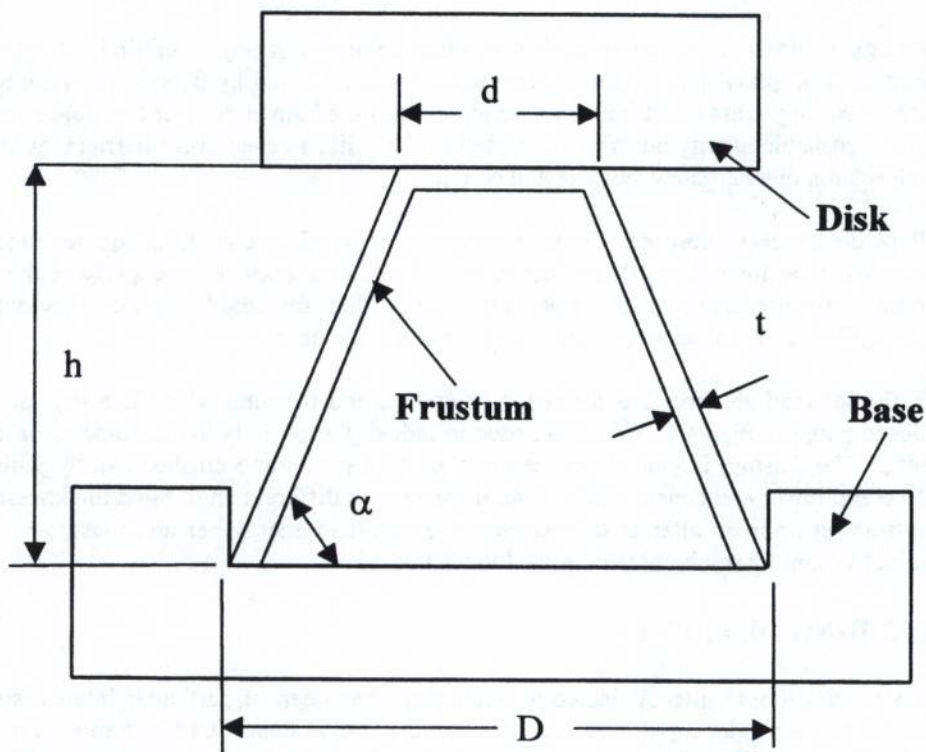

Figure 1: The experimental set up and the specimen.

The possibility of re-using the inverted frusta was investigated [18]. Several experimental tests were conducted for inversion and then re-inversion of inverted frusta. Other studies related to crushing of frusta including axial crushing of spun aluminium frusta [19,20], axial crushing of frusta with square cross-section [21], axial crushing of composite frusta [22] and axial crushing of constrained frusta [23].

The above studies deal with axial crushing (or inversion) of frusta between two parallel plates. However, in this paper the boundary condition is changed to be free at the upper small capped-end and fixed at the large lower open-end. Although El-Sobky, Singace and Petsios [23] studied axial crushing of constrained frusta with top-constrained, base-constrained and fully constrained, they didn't report any special crumpling pattern other than the regular crushing mode.

During axial crushing of capped-end frusta [20] it was noticed that by constraining the lower end for frusta with specific angles, the mode of deformation is changed into what might be called folding-crumpling mode and the crushing force and hence the absorbed energy is significantly high. In what follows, results of experimental work conducted on folding-crumpling of capped-end machined aluminium frusta are presented.

EXPERIMENTAL

The folding-crumpling mode was first observed when aluminium spun frusta were crushed axially for frustum angle (α) greater than 60°. Then aluminium specimens were machined from commercial aluminium bar with 80-mm diameter, see Table 1. The frusta had different thicknesses, 1-mm to 3-mm, and different angles (α) of 65° to 80°. They were subjected to subsequent heat treatment to get rid of the locked-in residual stresses developed during machining. The programme involved using different sizes of aluminium frusta (4 different angles and 4 different thicknesses) for folding-crumpling tests. The average flow stress of the specimen is equal to 85MPa.

Tests were conducted using 10-ton Instron Universal Testing machine (UTM) and special jig was manufactured and utilized. The jig consisted of a thick solid disk, to be placed on the top of the specimen, and a base disk with recess. The lower open end of the frustum was constrained from radial movement in the base, see Figure 1.

Table 1: Dimensions of the specimens used.

Sp. No.	α (deg)	D (mm)	d (mm)	t (mm)	h (mm)	Mass (g)
6515	65	75	20	1.5	58	41.75
7015	70	75	20	1.5	75	47.25
7510	75	75	20	1.0	102.6	32.10
7514	75	75	20	1.5	102.6	55.02
7515	75	75	20	1.5	102.6	54.94
7516	75	75	20	1.6	102.6	63.80
7520	75	75	20	2.0	102.6	76.57
7525	75	75	20	2.5	102.6	98.01
7530	75	75	20	3.0	102.6	119.7
8015	80	75	20	1.5	156.0	81.93

RESULTS AND DISCUSSION

In this section, details of the experimental load-displacement curves for folding-crumpling mode are presented. As mentioned above the folding-crumpling mode was observed for frusta with angle greater than 60°. Typical force-deformation diagram is given first and the effects of angle of frustum and wall thickness are discussed later.

In Figure 2, load-displacement curve is presented for a frustum loaded axially. The specimen (Specimen 7516) is made of aluminium with thickness t=1.6 mm, small diameter d=20 mm, large diameter D=75 mm, mass m=63.80 g and angle α=75°. Although the experimental curves reported in the literature for axial crushing of frusta are fluctuating, in nature, but not in this systematic way with increase in crushing force and increase in wavelength of the plastic zone (lobe). The average crushing force is 23.21 kN and the absorbed energy is 1972 J giving the energy density to be 30.91 J/g. The load increases from zero to the first peak in Figure 2, which represents the instability point.
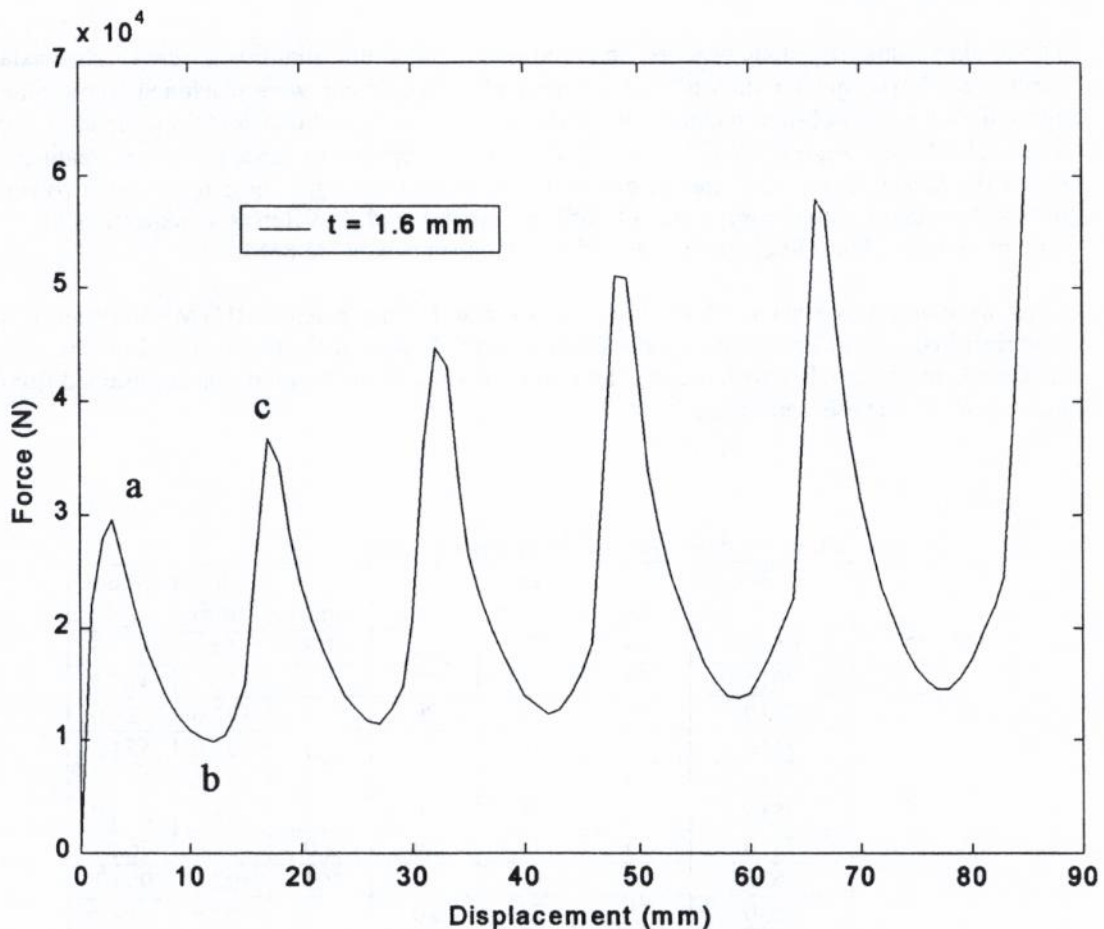

Figure 2: Load-displacement curve for Specimen 7516.

At this point, formation of the first lobe started. This lobe represents the active plastic zone close to the upper end. Four plastic hinges, and three shells represent this lobe as shown in Figure 3. As the load decreases from point a to point b in Figure 2, the upper shell of the lobe tends to collapse inward while the lower one tends to collapse outside, as shown in drawing j of Figure 3. Further deformation of the frustum straighten the middle shell of the lobe while stretch the upper one inward as a small tube and the lower one outward as a large tube, as shown in sketch k of Figure 3. Note that middle shell is not as large as shown here, but this is a simplified model. Also, plastic hinges keep moving within the lobe. Drawing k in Figure 3 represents point b in Figure 2 which is the lowest point between points a and c. At this point load starts increasing at low rate for few millimetres while the plastic hinges in the middle became one and the three shells became two as shown in sketch l of Figure 3. The sudden increase in the load is attributed to the upper jaw touches the undeformed part of the frustum where the first and the fourth plastic hinge became in one plane. The first lobe is completely formed and the second lobe starts forming as shown in sketches m and n of Figure 3. Note that the height of the second lobe is larger than the first one. The test continues in the same manner of lobe formation inside each other till the end.

Figure 4 shows frames of photos were taken during folding-crumpling of different specimen (Specimen 7530). These frames can help to understand the mechanism given in Figure 3. If the frustum were free (unconstrained) to move radially at its lower end, then the deformation mode would be outward (inversion) flattening as reported in literature [20]. However, because of the bottom-constrained boundary condition, deformation takes place at the top side. In addition, very thin frustum would deform in diamond mode of deformation as reported in literature [23]. Lobe starts at a peak and ends at the successful peak. Figure five shows top and bottom views of Specimen 7516 at the end of the experiment. Note that number of inside lobes in Figure 5 represents number of troughs in Figure 2 while number of outside lobes represents number of peaks in Figure 2.

Angle effect

The folding-crumpling mode depends heavily on the angle of frustum. Frusta with free upper capped-end and fixed lower end with different angles (30° to 80°, at 5° step) were crushed axially. The folding-crumpling mode was seen for frusta with angle greater than 65°. Figure 6 shows the load-displacement curves for identical frusta but different angles. One can see clearly that the pattern of folding-crumpling mode (seen above in Figure 2) is very clear for angle $\alpha=75°$. Other angles show similar behaviour especially at angle $\alpha=80°$. Frusta with small angles tend to crush into diamond mode of deformation and number of diamond lobes decreases with the increase in thickness. On the other hand, cylinders (frusta with 90° angle) tend to buckle into concertina deformation mode if the cylinder is thick and diamond deformation mode if the cylinder is thin. Folding-crumpling mode of frusta is similar to concertina mode for cylinder with few differences. First, the pattern is more uniform in folding-crumpling when compared to concertina, i.e. fewer irregularities, see reference [8]. Second, generally the load increases with deformation while it is constant for concertina. Third, formed lobes are above each other in concertina mode while they are inside each other in folding-crumpling mode. There is no clear clue why folding-crumpling mode takes place for particular angle of frusta other than the given geometry (diameters and thickness) of the frusta and the applied boundary condition (free upper capped-end and fixed lower open-end).

Thickness change

Effect of thickness change is seen in Figure 7 that shows the load-displacement curves for four identical frusta with the same angle ($\alpha=75°$) but different thickness. The folding-crumpling mode is seen to be independent of the thickness. However, one can notice that the number of lobes increases with the decrease in thickness. Number of lobes decreases from six lobes for t=1.5 mm to four lobes for t=3 mm. Also, some irregularities (crushing pattern) were seen during testing of specimen with t=2 mm, but the overall pattern is folding-crumpling mode. Thicker frusta required longer wavelength, thus fewer number of lobes. The longer wavelength is needed to stress thick cross

sectional area up to the yield strength. Stress is developed by bending of the plastic hinges shown in Figure 3.

Energy density

It was mentioned earlier that the objective of carrying out this study is to maximize the energy absorbed per unit mass. Figure 8 illustrates clearly the effect of high density on this mode when compared to other modes reported in the literature, mainly crushing and inversion. Specific energy, calculated by dividing the energy absorbed by the mass of the absorber, is plotted against the angle of the frustum, α. For all cases shown, the folding-crumpling mode absorbed more energy when compared to other modes. At angle $\alpha=75°$ the energy absorbed difference is maximum because folding-crumpling mode is seen to be perfect at this angle.
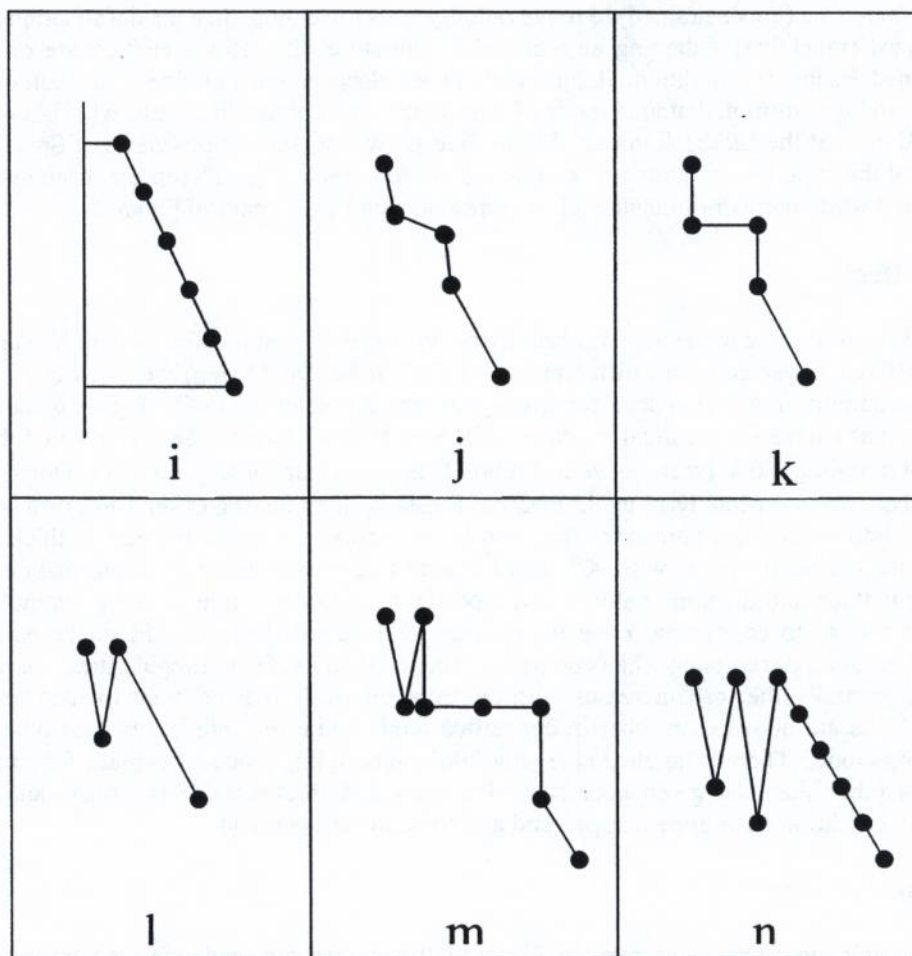
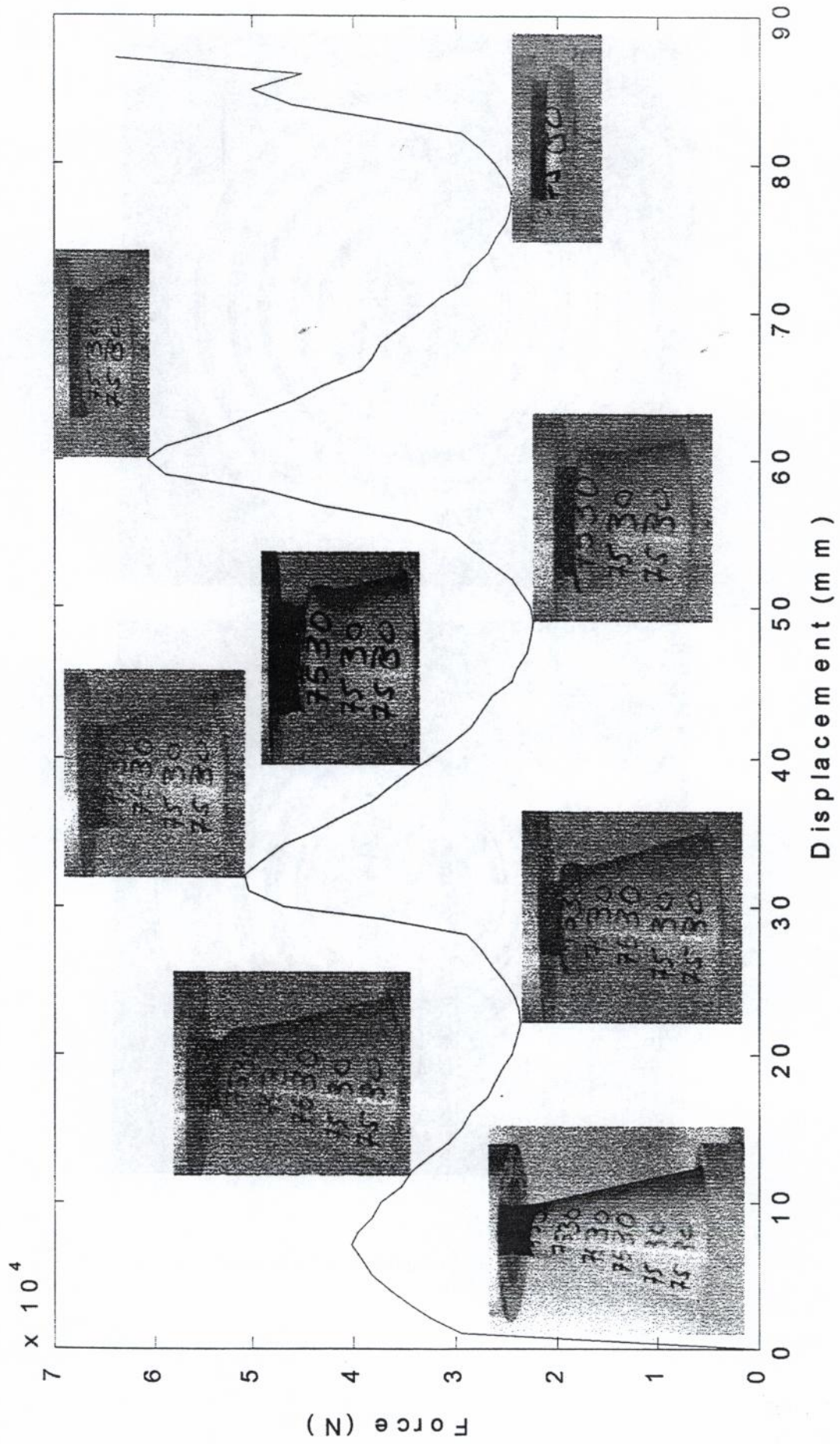Figure 3: Mechanism of the folding-crumpling mode of deformation.

Figure 4: Load-displacement curve for specimen 7530. Each frame is taken at the proper point shown in the curve.
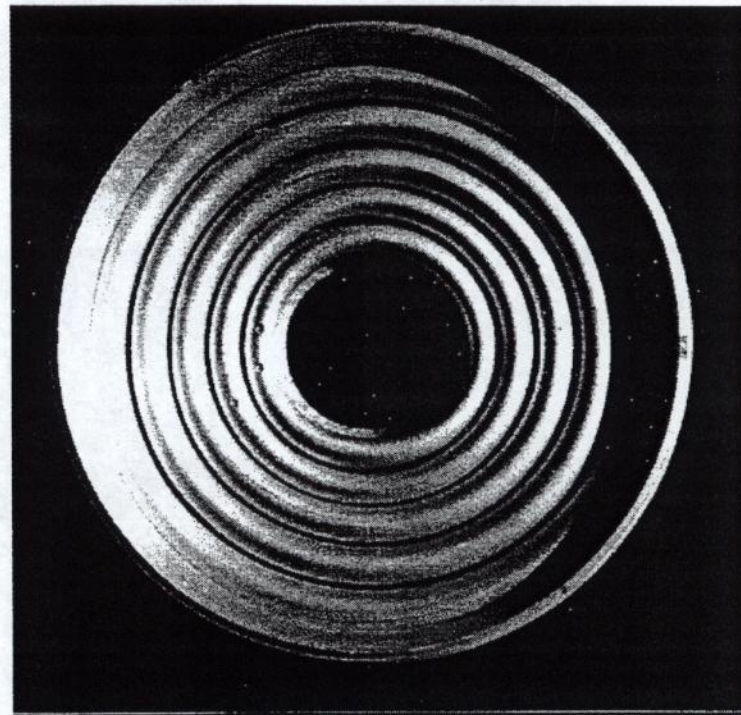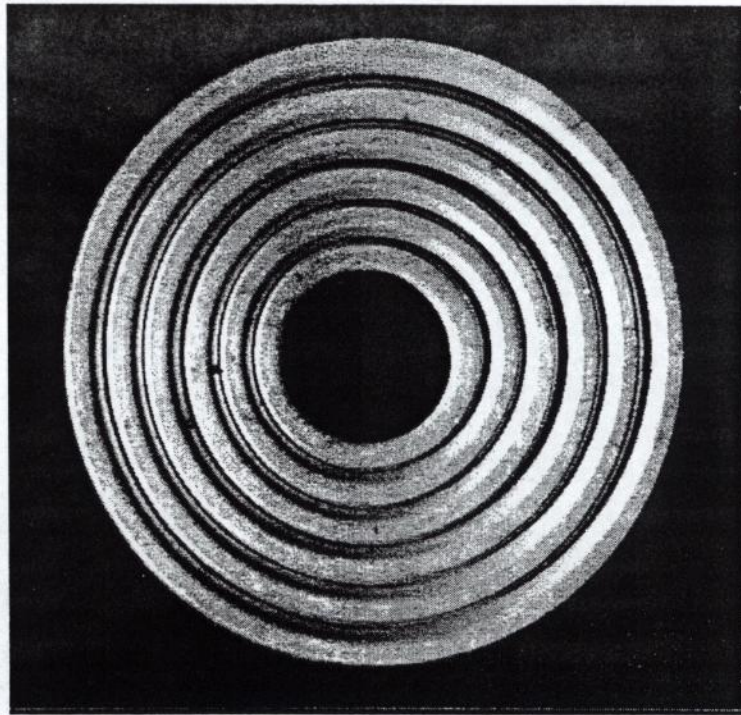
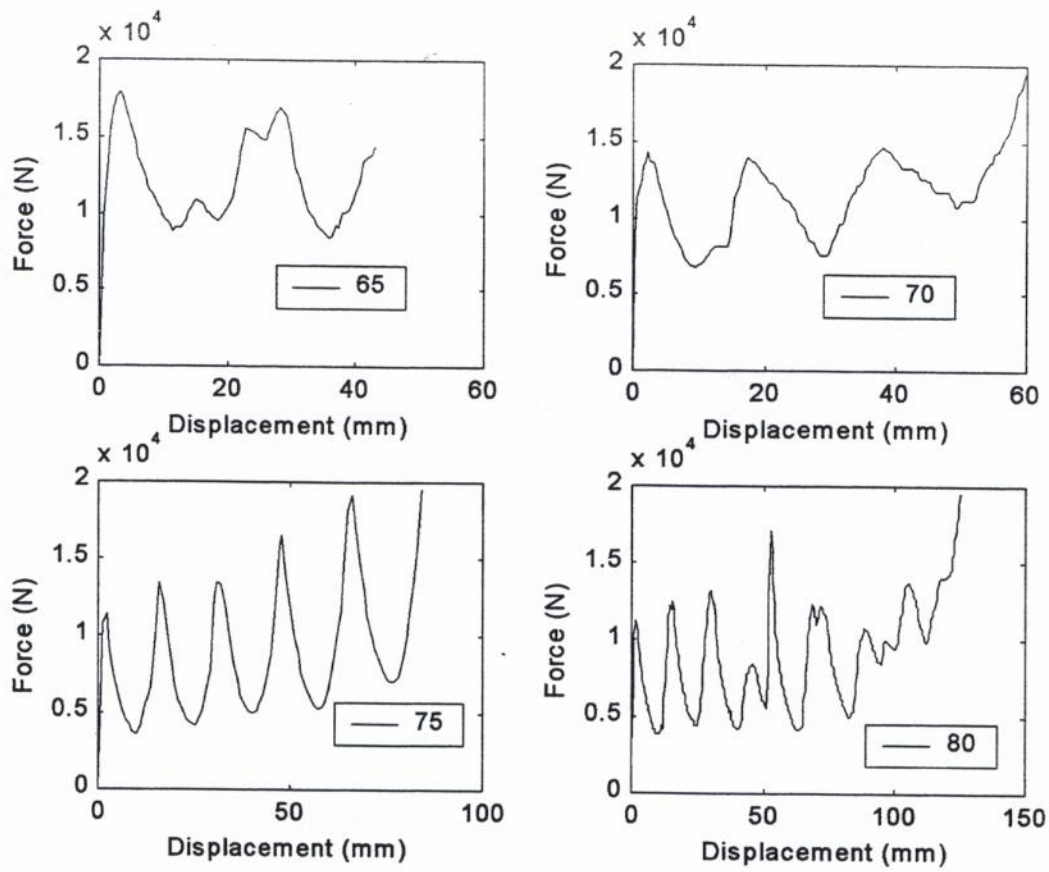Figure 5: Top (at top) and bottom views of specimen 7516.

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Figure 6: Load-displacement curves for identical frusta but with different angles.
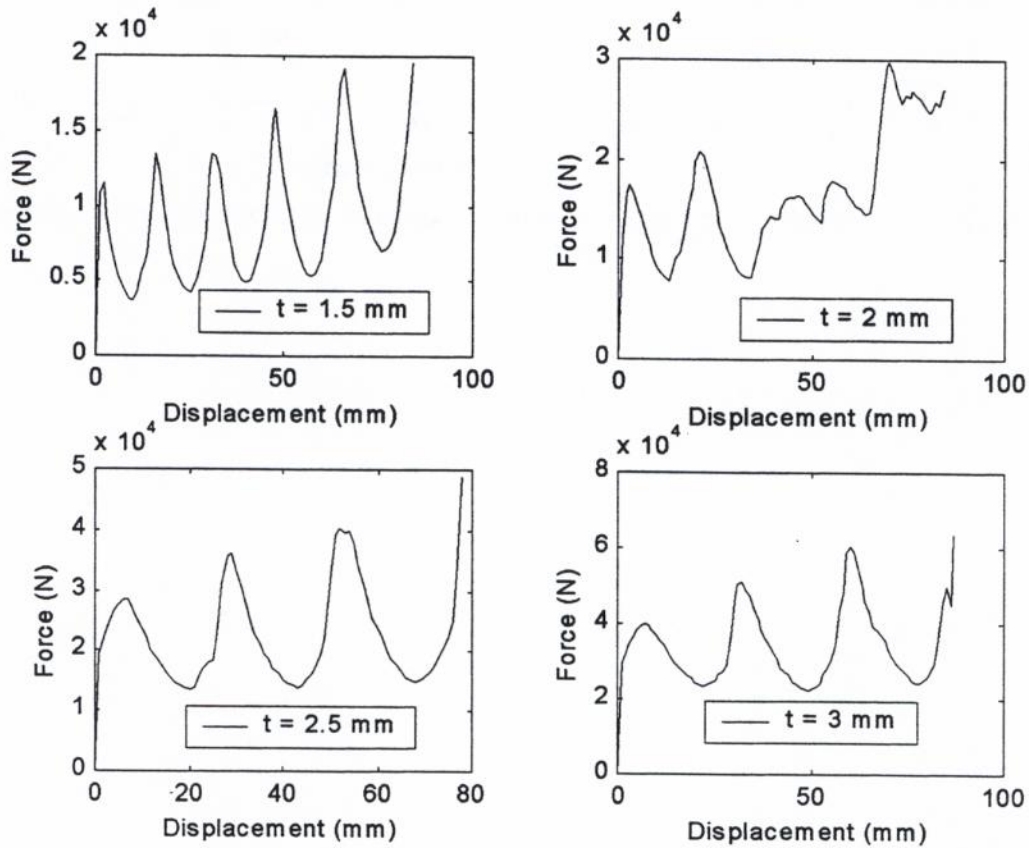
Figure 7: Load-displacement curves for frusta with the same angle but different thickness.
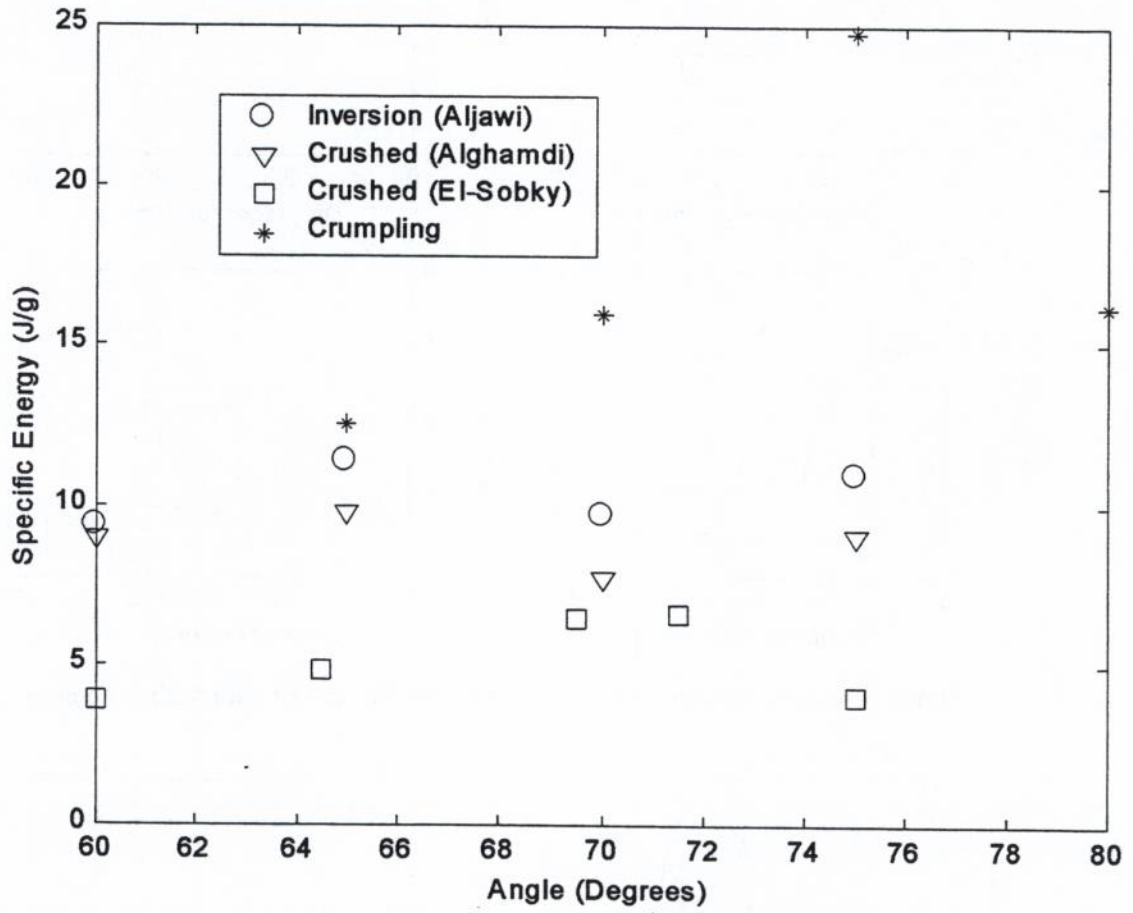
Figure 8: Specific energy as a function of angle of frustum for different modes of deformation.

CONCLUSIONS

Researchers have studied crushing of frusta for many years. However, this paper discusses a new mode of deformation called folding-crumpling mode that was seen for frusta with large angles in the region of 75° for free-fixed boundary conditions. This innovative mode absorbed large energy when compared to other reported modes. The increase in the energy density is attributed to the large plastic deformation that the frustum undergoes in this mode. This plastic deformation is seen in terms of more plastic hinges, larger angle of plastic bending and more stretching work. In conclusion it is recommended to use frusta with $\alpha=75°$ and with free-fixed boundary condition.

ACKNOWLEDGEMENT

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